Ibn Al-Haytham (Alhazen)

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Among the mathematicians of classical Islam, few are as famous as al-Ḥasan ibn al-Ḥasan ibn al-Haytham (Alhazen in the Latin West). A physicist and astronomer as well as mathematician, he quickly gained a wide reputation, first in Arabic, in the Islamic East as well as the Islamic West, and then from the translations of his works in optics and astronomy into Latin, Hebrew, and Italian.

But his renown, completely justified by the importance of his contributions and especially of the scientific reforms accomplished in them, contrasts singularly with the paucity of information we have on the man, his teachers, or his scientific milieu. Also, the significance of his works surrounded the man with the aura of a legend. Sources available to us consist of narratives recounted by ancient bibliographers where legend becomes mixed up with the rare historical evidence. These same narratives are precisely what modern bibliographers continue to reproduce partially or totally until today. After a critical reading of these sources, very little information remains: born in Iraq, most likely in Bassorah, sometime in the second half of the tenth century, Ibn al-Haytham arrived in Cairo, under the reign of Faṭimid al-ḥākim. He proposed a hydraulic project to control the waters of the Nile, but it was rejected by the Caliph. He continued to live in Cairo until his death, after 432/1040.

From the thirteenth century until today, biographers have confused al-Ḥasan ibn al-Haytham with Muḥammad ibn al-Haytham, a philosopher and theorist of medicine who lived in Baghdād at the same time. This confusion, due undoubtedly to similarity of the two names of these contemporary authors, is serious as it brings into question the authenticity of certain writings attributed to al-ḥasan ibn al-Haytham.

Biobibliographers, notably al-Qifṭī, cite 96 titles of Ibn al-Haytham, not all of which have survived. Half of his writings are in the field of mathematics, 14 on optics, including the authoritative and voluminous *Kitāb al-Manāẓir* (Book of Optics), 23 on astronomy, two in philosophy (one on the *Place* and the other on the *Indivisible Part*), three on statics and hydrostatics, two on astrology, and four on various other topics. This accounting shows clearly that Ibn al-Haytham grappled with all the mathematical sciences of that time, or at least the most advanced part of this discipline. We will see that he was always at the leading edge of research or at the culmination of one tradition and the beginning of a new period. It is precisely this quality which distinguishes his contributions. Ibn al-Haytham lived at a privileged time, his work following a century of intense research in these fields by eminent scholars such as the Banū Mūsā, Thābit ibn Qurra and his grandson Ibrāhīm ibn Sinān, al-Qūhī, and Ibn Sahl, to name a few. We will now briefly examine the principal aspects of his research.

Mathematics

Ibn al-Haytham's mathematical research was particularly in the field of geometry and not of algebra. Ancient biobibliographers attributed a boọk on algebra to him, but it has not survived. From the outset, geometers wanted to combine closely the study of the positions of figures and their metric properties: in other words, to combine the geometry of Apollonius with that of Archimedes. This combination is not a static synthesis, but a new organization of geometry which possessed a real heuristic value. Already initiated by al-Ḥasan ibn Mūsā and followed by Thābit, this work led to the study of geometric transformations and of projective methods. It was this work which Ibn al-Haytham developed further in his own geometrical studies.

The contributions of Ibn al-Haytham in geometry can be divided into several groups, the most important of which are in infinitesimal mathematics and the theory of conic sections and their applications. He composed 12 treatises on infinitesimal mathematics and then on conic theory. To those can be added a third area in which Ibn al-Haytham takes up several problems relating to the foundations of mathematics and their methods in his treatise *Maqāla fī'l-taḥlīl wa ʻl-tarkīb* (On Analysis and Synthesis), his *Kitāb fī al-ma ʻlūmāt* (On the Known Things), his *Sharḥ Uṣūl Uqlīdis fī ʻl-handasa wa ʻl-ʻadad wa talkhīṣuhu* (Commentary on the *Elements* of Euclid), and his *Kitāb fī Ḥall shukūk Kitāb Uqlīdis fi ʻl-uṣūl wa-sharḥ maʻānīh* (Solutions to Doubts) again concerning Euclid. In these books, he deals as much with the constitution of a new discipline, a kind of proto-topology, as with the theory of the demonstration within the difficulties raised by the fifth postulate of Euclid, or with the theory of parallels. Ibn al-Haytham also edited an important paper on number theory, four treatises on arithmetic, and the same number on practical geometry.

Of the 12 papers on infinitesimal mathematics, only seven have survived. The first three are devoted to the study of lunes and the quadrature of a circle. Note that the calculation of the area of lunes involves the calculation of sums or differences of areas of sectors or of triangles, the comparison of which has recourse to that of the ratio of angles or of the ratio of segments. In the most important of the three papers, Ibn al-Haytham begins by setting up four lemmas, the results of which demonstrate the role of the function *f*, defined as

$$f(x) = \frac{\sin^2 x}{x}$$

in the study of lunes.

In his study *Tarbīʻal-dāʻira* (On the Quadrature of a Circle), he examines the relationship between proving the existence of a magnitude or a property and the question of effectiveness of its construction.

Other treatises on infinitesimal mathematics deal with the volume of a solid curve: *Misāḥat al-mujassam al-mukāfiʻ* (The Measurement of a Paraboloidal Solid) and *Misāḥat al-kura* (The Measurement of a Sphere). In calculating the volume of a paraboloid, Ibn al-Haytham deals rapidly with the volume of a revolving paraboloid, which had already been studied by Thābit ibn Qurra and al-Qūhī. He then moves on to his own invention: how to calculate the volume of a paraboloid obtained from the rotation of a parabola around its ordinate. He shows that this volume is 8/15 of the volume of the circumscribed cylinder. His calculation is equivalent to that of the integral

$$\pi \int_a^b k^2(b^2 - 2b^2y^2 + y^4)\mathrm{d}y = \frac{8}{15}\pi k^2 b^5 = \frac{8}{15}V,$$

with *V* being the volume of the circumscribed cylinder.

Ibn al-Haytham proceeded in this study with the help of the method of integral sums, which he also applied in calculating the volume of a sphere. In order to do this calculation, Ibn al-Haytham generalized the proposition *X*-1 of Euclid's *Elements*. He devoted the seventh paper in this group to that. This group includes an important treatise devoted to isoperimetric and isepiphane problems. It was the most advanced mathematical work of its time and for several centuries following. In it, to study these *extrema*, he had to undertake the first substantial research on the theory of a solid angle. Moreover he combined both a projective method and an infinitesimal method.

Ibn al-Haytham's second group of mathematical writings dealt with the theory of conic sections. He was well acquainted with the *Conics* of Apollonius and had copied them in his own hand, so he knew that, in Greek, the eighth and last book was lost. He tried to reconstruct the book according to the indications of Apollonius. In addition to his writings on conics, he applied the theory of the intersection of conics to the resolution of problems which cannot be constructed with a compass or ruler, problems either passed down (for example, the regular heptagon) or posed by him (for example, the solution of a solid arithmetic problem). Ibn al-Haytham was one of the first mathematicians who insisted on demonstrating the existence of the point of intersection of two conics in these last examples.

It is impossible in this space to explicate the mathematical results of Ibn al-Haytham's work. But let us simply note his expression of what is called Wilson's theorem, and the converse of Euclid's theorem for perfect numbers.

Indeed, in the course of solving the problem called the Chinese Remainder, he stated Wilson's theorem, which can be written as:

n is prime

$$(n - 1)! - 1(\mathrm{mod}\, n).$$

As for the converse of Euclid's theorem of perfect numbers, he tried to show that any even perfect number is in Euclidean form, in other words in the form $2^p(2^{p+1} - 1)$ with $(2^{p+1} - 1)$ prime.

Optics

A brief look at the work of Ibn al-Haytham on optics reveals not only its revolutionary nature but also its comprehensiveness, touching all the known branches of optics: optics in its proper sense in his *Book on Optics* and his *Discourse on Light*; catoptrics, notably burning mirrors (parabolic and spherical burning mirrors); dioptrics, in *al-Kura al-muḥriqa* (The Burning Sphere); and meteorological optics in *Daw al-qamar* (The Light of the Moon), *Aḍwā al-kawākib* (The Light of the Stars), *Fī Ṣūrat al-kusūf* (On the Shape of the Eclipse) and *al-Hāla wa-qaws quzaḥ* (The Halo and the Rainbow). With this extension, Ibn al-Haytham modified the meaning of optics. Optics is not any more reduced to a theory of direct vision, a geometry of the gaze with which a theory of vision is associated, but also bears significantly on the theory of light, its propagation, and its effects as a material agent. This leads us to the revolution accomplished by Ibn al-Haytham in optics and more generally in physics.

Ibn al-Haytham sought to bring about a program of reform, which led him to take up a whole range of different problems. The basic aspect of this reform was to clarify the difference between the conditions of the propagation of light and the conditions of the vision of objects. This led, on the one hand, to giving physical support to the rules of propagation – making a firm mathematical analogy between a mechanical model of the movement of a solid ball thrown against an obstacle and that of light – and on the other hand to proceeding geometrically at all times, both by observation and by experimentation. Optics consisted henceforth of two parts: one, a theory of vision and the associated physiology of the eye and psychology of perception, and the other, the theory of light to which are linked geometric optics and physical optics. The organization of the *Optics* reflects this new situation: there are chapters devoted entirely to propagation, such as the third chapter of the first book and books IV to VII; others deal with vision and related problems. This reform also resulted in the emergence of new problems, such as Alhazen's problem in catoptrics, the examination of the spherical lens and the spherical diopter, not only as burning instruments but also as optical instruments in dioptrics, and to experimental control, viewed as much as a general practice of investigation as the norm of a proof in optics, and more generally in physics. Let us now take a quick look at how this reform in optics was carried out.

Ibn al-Haytham rejected any doctrine of a ray stemming out from the eye, called a visual ray, in order to defend the intromissionist theory of visible forms. But unlike the intromissionists of antiquity, he did not believe that objects sent off "forms" or totalities which emanated from the visible under the effect of light. He saw them rather as forms reducible to their elements: a ray emanating toward the eye from every point of a visible object. Looked at thus, the eye becomes a simple optical instrument. Ibn al-Haytham then explained how the eye perceives the visible with the help of its rays emitted from all points. In the *Optics* he devotes the first three chapters to the foundations of this theory. In the three following chapters, he deals with catoptrics. The seventh and last chapter is devoted to dioptrics. His theories rest on two qualitative laws of refraction and on several quantitative rules, all controlled experimentally with the help of an instrument which he designed and built himself. The two qualitative laws, known to his predecessors Ptolemy and Ibn Sahl, can be stated as follows:

1. The incident ray, the normal at the point of refraction, and the refracted ray are in the same plane; the refracted ray gets closer (respectively, far away) from the

normal, if light passes from a milieu less (respectively, more) refringent to a milieu more (respectively, less) refringent

2. The principle of inverse return

Instead of pursuing the path opened up by his predecessor Ibn Sahl, Ibn al-Haytham returned to a study of angles in order to establish the quantitative rules. He devoted a substantial part of the seventh book to a study of the refracted images of an object, notably if the surface of separation of two milieux is either planar or spheric. It was in the course of this study that he fixed his attention on the spherical diopter and the spherical lens. He returned to the spherical lens in his treatise *On the Burning Sphere*, one of the high points of research in classical optics, in order to improve upon certain results that he had already obtained in his *Optics*. This treatise was the first deliberate study on the spherical aberration for parallel rays falling on a glass sphere and giving off two refractions.

Astronomy

By their number, their thematic variety, the power of the analysis they show, and by their results, the works of Ibn al-Haytham in astronomy yield nothing to his works in mathematics or optics. It should be noted only that an elementary treatise of Muḥammad ibn al-Haytham, the *Commentary on the Almagest* is often erroneously attributed to Ibn al-Haytham. The attribution of the book *On the Configuration of the Universe* to him is also doubtful. The authentic works of Ibn al-Haytham have not yet been seriously studied, *a fortiori*, apart from a few rare and particular contributions, such as the *Samt al-ibla bi-al-ḥisāb* (Determination of the Direction of Mecca by Calculation). It remained for subsequent astronomers, notably al- ʿUrḍī, one of the founders of the school of Marāgha, to recognize their debt to Ibn al-Haytham's book *al-Shukūk ʿalā Baṭlamyūs* (Doubts on Ptolemy). Before assessing his contribution in astronomy, we must wait until his books have received the editing and study that they deserve.

The impact of the work of Ibn al-Haytham varies according to the field. In mathematics, his influence can be seen in the works of Ibn Hūd, al-Khayyām, Sharaf al-Dīn al-Ṭūsī, and al-Samaw'al, among others. But we do not know anything of successors who might have tried to follow up on his research on lunes, the solid angle, or the measurement of figures and solid curves. In optics, the Latin translation of his *Optics* (under the title *Perspectiva* or *De Aspectibus*, reedited in 1572 under the title *Opticae Thesaurus*) and his treatise *On Parabolic Burning Mirrors* provided a basis of research for centuries of scholars such as Witelo, Roger Bacon, J. Peccham, Frederick of Fribourg, Kepler, and Snell, among many others. In Arabic, there is the commentary of Kamāl al-Dīn al-Fārsī. Finally in astronomy, there is the work of al- ʿUrḍī which shows the influence of his work. It is too soon to measure the impact of the writings of Ibn al-Haytham on his successors in this field also, but they appear to be immense.

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